In the E373 experiment at KEK-PS, we have located nearly $2 \times 10^4$ stopping events of $\Xi^-$-hyperon candidates in nuclear emulsion. Among these, the identification of the $\Xi^-$ hyperon was performed with the constant sagitta (CS) method, by measuring multiple Coulomb scattering for 695 clear stopping events, $\sigma$-stop, with emission of nuclear fragments due to negatively charged hadrons. With use of a Geant4 simulation, the parameters for the CS method were optimized and we obtained the number of real $\Xi^-$ stopping events to be $432.3 \pm 7.6_{-14.0}^{+0.0}$ with 3.2% systematic error by contamination by the $\Sigma^-$ hyperon. We have classified the events for $\Xi^-$-hyperon capture by light (C, N, O) and heavy (Ag, Br) elements in the emulsion. The trapping probabilities of two $\Lambda$ hyperons for light and heavy nuclei were found to be 5.0\(\pm\)1.7\% and 4.2\(\pm\)1.4\%, respectively, for $\sigma$-stop via $\Xi^-$ hyperon capture at rest in the emulsion. For at least one $\Lambda$ trapping, the probabilities were 69.4 \(\pm\) 8.1\% for light nuclei and 51.1 \(\pm\) 5.7\% for heavy nuclei. These results allow us to present the trapping probabilities of $\Lambda$ hyperons via $\Xi^-$-hyperon capture at rest in emulsion for the first time, because the previous E176 experiment showed the lower limits due to fewer statistics.
Letter

Trapping probability of strangeness via \( \Xi^- \) hyperon capture at rest in nuclear emulsion

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In the E373 experiment at KEK-PS, we have located nearly \( 2 \times 10^4 \) stopping events of \( \Xi^- \) hyperon candidates in nuclear emulsion. Among these, the identification of the \( \Xi^- \) hyperon was performed with the constant sagitta (CS) method, by measuring multiple Coulomb scattering for 695 clear stopping events, \( \sigma \)-stop, with emission of nuclear fragments due to negatively charged hadrons. With use of a Geant4 simulation, the parameters for the CS method were optimized and we obtained the number of real \( \Xi^- \) stopping events to be \( 432.3 \pm 7.6_{-14.0}^{+0.0} \) with 3.2% systematic error by contamination by the \( \Sigma^- \) hyperon. We have classified the events for \( \Xi^- \) hyperon capture by light (C, N, O) and heavy (Ag, Br) elements in the emulsion. The trapping probabilities of two \( \Lambda \) hyperons for light and heavy nuclei were found to be \( 5.0 \pm 1.7\% \) and \( 4.2 \pm 1.4\% \), respectively, for \( \sigma \)-stop via \( \Xi^- \) hyperon capture at rest in the emulsion. For at least one \( \Lambda \) trapping, the probabilities were \( 69.4 \pm 8.1\% \) for light nuclei and \( 51.1 \pm 5.7\% \) for heavy nuclei. These results allow us to present the trapping probabilities of \( \Lambda \) hyperons via \( \Xi^- \) hyperon capture at rest in emulsion for the first time, because the previous E176 experiment showed the lower limits due to fewer statistics.

Subject Index D25

1. Introduction Investigation of the double-strangeness system provides information about the \( \Lambda-\Lambda \) and \( \Xi-N \) interactions, which are very important in understanding baryon–baryon interactions in the SU(3)\(^f\) symmetry scheme. To produce double-\( \Lambda \) hypernuclei and \( \Xi \) hypernuclei, \( S = -2 \) nuclei, the at-rest capture reaction of \( \Xi^- \) hyperons was utilized in nuclear emulsion for a long time, where \( \Xi^- \) hyperons were created via quasi-free \( “p”(K^-, K^+)\Xi^- \) reactions. Since the \( Q \)-value for the conversion reaction of \( \Xi^- + p \rightarrow \Lambda + \Lambda \) is at most 28 MeV, the path length of double-\( \Lambda \) hypernuclei is very short, e.g. several \( \mu \)m, in the emulsion. From this point of view, an emulsion with high spatial resolution better than 1 \( \mu \)m would be very useful to detect the production and decay processes of \( S = -2 \) nuclei.

At the capture point of \( \Xi^- \) hyperons, \( S = -2 \) nuclei were formed through the above conversion reaction in a certain possibility. It is very important to get the trapping probability for two \( \Lambda \) hyperons, because it can be affected by the strength of the \( \Xi-N \) interaction and the formation mechanism of...
Although little is known about the trapping probability, an experimental result has been obtained by E176 at KEK-PS.

The E176 experiment at KEK-PS, an emulsion-counter hybrid experiment, was performed with 1.66 GeV/c K⁻ beam-induced “p”(K⁻, K⁺)Ξ⁻ reactions by high-momentum K⁺ tagging. The emulsion acted not only as a detector of S = −2 nuclei, but also as a production target of Ξ⁻ hyperons; thus we were able to recognize the background of the capture reaction of Ξ⁻ candidates with a precise study of the kinematics at (K⁻, K⁺) reaction vertices. Based on the stop events of certain Ξ⁻ hyperons, it was reported that the lower limits of the probabilities for two-Λ trapping were 4.8% and 1.7% for light (C, N, O) and heavy (Ag, Br) elements at the 90% confidence level, respectively [1].

To get 10 times higher statistics of E176, we performed the E373 experiment at KEK-PS, also with a 1.66 GeV/c K⁻ beam, and obtained ∼700σ-stop events. However, the amount of background for the certain capture of Ξ⁻ hyperons was not known. The main reason for this was that the “p”(K⁻, K⁺)Ξ⁻ reaction vertices were in the diamond target but not in the emulsion [2], so it was not possible to check the kinematics sufficiently, due to scattering in the target.

To obtain the trapping probabilities, we have developed a method to identify the Ξ⁻ hyperon in σ-stop events by measuring multiple Coulomb scattering with respect to almost-straight beam tracks. We applied this method to Ξ⁻ candidate tracks of σ-stop events with checking by known double hypernuclei, and obtained the trapping probabilities of two Λ hyperons for both light and heavy elements in the emulsion.

In Sect. 2, we will briefly review the E373 experiment; an evaluation of the method will be introduced in Sect. 3. Trapping probabilities will be discussed in Sect. 4.

2. The E373 experiment at KEK-PS  The KEK-PS E373 experiment was performed with two main purposes. The first one is to identify the Λ–Λ interaction energy by detection of the ground-state double-Λ hypernucleus. The energy should set a limit on the mass of the H dibaryon [3], which was predicted to be stable under the strong interaction. The other purpose is to study energy states of the nuclei absorbing Ξ⁻ hyperons via analyses of twin single-Λ hypernuclear events, and then the information can provide for the presence of the Ξ hypernucleus.

To accomplish the above two purposes, one of the key points is to obtain 10 times more double hypernuclei than E176 by separating the production target for Ξ⁻ hyperons and for double hypernuclei, under the limited K⁻ content in the beam at KEK-PS. We utilized a diamond for production of Ξ⁻ hyperons, because its effective proton number is higher than the elements composing the emulsion. The size of the diamond target was 2 × 2 × 3 (beam direction) cm³. The diamond target was located just upstream of a scintillating-fiber (Scifi)-bundle tracker [4], which was set 0.5 mm upstream of the emulsion stack. The produced Ξ⁻ hyperon loses its kinetic energy in the diamond target and the emulsion, so the stopping of Ξ⁻ hyperons could be increased before their decay in the emulsion. Details of the experimental setup are introduced in Ref. [2] and in unpublished work by K. Nakazawa et al.

The Ξ⁻ candidates stopping in the emulsion were nominated by the checking absence of a thin track like π⁻, the decay daughter of the Ξ⁻ hyperon, or a thick track of the Ξ⁻ hyperon through the stack with an event-by-event image of the Scifi-block tracker [2] located downstream of the stack.

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1 At least one charged nuclear fragment is emitted from the end point of the Ξ⁻ hyperon candidate track. This emission shows the capture of negatively charged particles.
The emulsion stack consisted of one thin-coated emulsion and 11 thick-coated emulsion sheets. The thin-coated sheet was scanned for detection of the $\Xi^-$ hyperon candidate track predicted by the Scifi-bundle tracker. The detected candidate tracks were followed downstream in the thick-coated sheets to their end points.

The end points of incident $\Xi^-$ candidate tracks into the emulsion were categorized according to the following six types: (1) OUT (escaping from the emulsion stack), (2) Stop ($\sigma$-stop and $\rho$-stop\(^2\) in the emulsion), (3) Beam int. (originating from beam interaction), (4) Secondary int. (secondary interaction with the emulsion nuclei), (5) Lost (lost in the gap or support film of the emulsion sheet), and (6) Decay (decay into a pion-like thin track). The results are summarized in Table 1.

The scanned results for the track followings are also summarized with the results of E176 in Table 1.

In the E176 experiment, $K^+$ candidate tracks were followed back to the upstream and arrived at 797 ($K^-$, $K^+$) reaction vertices, where the momentum of outgoing $K^+$ mesons was $p_{K^+} \geq 1.0 \text{ GeV}/c$. Among these, $\Xi^-$ candidate tracks were detected at 555 reaction vertices. This discrepancy gave the mean-free path of the $\Xi^-$ hyperon inside the nucleus [5]. In the case of type (2) in E176, the number of Stop events was concluded to be $77.6 \pm 5.1 + 0.0 - 12.2$ (systematic error by contamination by the $\Sigma^-$ hyperon), where the $\sigma$-stop and $\rho$-stop events were $52 \pm 0.0 + 0.0 - 5.0$ and $25.6 \pm 5.1 + 0.0 - 7.2$, respectively, with kinematic correction by the data at the ($K^-$, $K^+$) vertex [1].

In the E373 experiment, we searched for $\Xi^-$ candidate tracks in the thin-coated sheet for 14 854 ($K^-$, $K^+$) events tagged by $p_{K^+} \geq 0.9 \text{ GeV}/c$. Among these, we followed 37 935 tracks as $\Xi^-$ candidates from 13 726 ($K^-$, $K^+$) events, where multiple $\Xi^-$ candidate tracks were detected for each tagged event in the first emulsion sheet due to insufficient prediction accuracy with the Scifi-bundle tracker. The numbers of $\sigma$-stop and $\rho$-stop events were 766 and 12 655, respectively. By no kinematic study at the ($K^-$, $K^+$) vertices, some background events can be contained in $\sigma$-stop events caused by negatively charged hadrons such as $\Sigma^-$ and $\pi^-$. In the $\rho$-stop events, almost all of them would be caused by positively charged particles such as protons in consideration of the ratio of $\rho$-stop to $\sigma$-stop events of $\sim0.5$ given by the E176 result.

To identify the $\Xi^-$ hyperon with multiple Coulomb scattering, sufficient track length from the end point in the emulsion is necessary, where the minimum length is 800 $\mu$m. Since the mass of the proton is close to that of the $\Xi^-$ hyperon, it was found to be impossible to separate nearly a few

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2 No charged nuclear fragments are emitted from the end point of the $\Xi^-$ candidate track. However, the stop of a positively charged particle also shows the topology of the $\rho$-stop.
Fig. 1. A schematic drawing of $\sigma$-stop coming from a negatively charged particle from the right. “×” marks show measured positions in 20 $\mu$m steps along the beam. The arrow for the $z$-direction points upstream of the stack. The cell length $t_i$ is calculated by Eq. (1). The “◦” marks assigned for $R_i$ show the position from the stopping point with the length of $R_i$.

hundred $\Xi^-$ hyperon from $1.2 \times 10^4$ protons in the preceding study [6]. Therefore we measured the multiple Coulomb scattering of the candidate tracks for 695 $\sigma$-stop events among 766 ones.

3. $\Xi^-$ identification with multiple Coulomb scattering measurement

3.1. Constant sagitta (CS) method

For a high-energy particle with velocity $\beta \sim 1$, the scattering angles measured after passing through a medium with fixed thickness and root-mean-square (RMS) values of the angular variation can be useful to estimate the momentum of the particle [7]. Regarding slow particles likely to stop, the CS method has been developed to uniformize scattering by changing the medium thickness, called cell length $t_i$, as in Eq. (1) [8]. The numerical values in Eq. (1) are empirical ones for particles in the emulsion introduced in Ref. [8]:

$$t_i = \left[ \sigma_0 \left( \frac{1}{0.003 48 \times K_s} \right) R_i^{0.58} \times Z^{0.16} \times M^{0.42} \right]^{\frac{1}{2}}. \tag{1}$$

We measured the track coordinates in 20 $\mu$m step from the $\sigma$-stop point upstream along the beam direction as “×” marks in Fig. 1(a). The track with a dizzy shape is approximated as a linearly interpolated line as shown in Fig. 1(b). In Eq. (1), $R_i$ is the range of track length from the stopping point, where $R_i$ starts at 500 $\mu$m from the stopping point to avoid the very dizzy region. The position with track lengths of $R_i$ is shown by “◦” marks in Fig. 1(b), where the relation between $R_i$ and $t_i$ is $R_{i+1} = R_i + t_i$. $\delta_i$, the so-called second difference, is the spatial length between the position of $R_{i+2}$ and the extrapolated line from the positions of $R_i$ and $R_{i+1}$ as shown in Fig. 1(b). The scattering constant, $K_s$, of an empirical parameter depending on the emulsion will be assigned to well present the RMS value of the measured second differences of a particle to be close to the value of the $\sigma_0$ (constant sagitta). $\sigma_0$ corresponds to the RMS value of a set of $\delta_i$, when we take suitable $t_i$ for a particle with a charge of $Z$ and a mass of $M$ by Eq. (1). $\sigma_0$ is set to the expected RMS value of the second differences. If we apply $M$ of the mass of the $\Xi^-$ hyperon to trajectories of the $\pi^-$ meson, the obtained RMS values of the second differences can be different from the expected $\sigma_0$ of real $\Xi^-$ hyperons.
Fig. 2. Distortion correction of track positions in an emulsion sheet. We measured track position in 20 μm steps along the z-axis; however, the steps are not shown in the figure to present the concept of the method for distortion correction. The “•” marks show the corrected positions.

Fig. 3. Obtained mean of the RMS of second differences as δrms for simulated Ξ− hyperons for several Ks values.

3.2. Assignment of scattering constant, Ks, with Geant4 simulation

In order to assign the value of Ks, we produced data samples of Ξ− and π− in the emulsion with the aid of a Geant4 simulation. The elemental composition of the emulsion of ET-7D type was used in the simulation and the density was 3.60 g/cm³. The data produced have no measurement errors. However, the track measurement must be done with finite accuracy under distortion due to non-uniform drying of emulsion gel during the sheet making.

To correct the distortion effect of the emulsion, we measured the positions of two beam tracks at each measured position of a Ξ− candidate track, where the beam would be recorded almost straight with a high momentum of 1.66 GeV/c. The distortion effects are presented as displacements from a straight line, as shown in Fig. 2, which shows the measured points as “×” marks for two beam tracks #1 and #2. The displacement vector has two elements of (Δxi, Δyi), which are changed along the z-axis. We measured the elements of displacement from the line made by two measured positions on both surfaces of the base, which is transparent film to support the emulsion layers, to the measured positions of track #1. These elements were applied to the measured positions of track #2, and obtained variabilities from a straight line as σx and σy for the x- and y-directions, respectively, with linear fitting. σx and σy were 0.246 ± 0.020 μm and 0.251 ± 0.025 μm, respectively.

We revised the data produced with measurement errors. By tentatively setting σ0 to 1.0 μm, we calculated RMS values of the second differences for the revised 20000 samples made by Geant4.
Fig. 4. RMS of second differences of the revised data samples with measurement errors for $\Xi^-$ and $\pi^-$ produced by the Geant4 package.

for the $\Xi^-$ hyperon in the range from 500 to 4000 $\mu$m by changing the $K_s$ value. The $K_s$ value is obtained by providing the mean of the RMS values, $\delta_{\text{rms}}$, for each sample to be close to the set $\sigma_0$ value of 1.0 $\mu$m. Figure 3 shows the $\delta_{\text{rms}}$ for several $K_s$ values. At the point of $\delta_{\text{rms}} = 1.0$, the most suitable $K_s$ value is obtained as 20.75.

We applied $K_s$ and $\sigma_0$ values of 20.75 and 1.0 $\mu$m, respectively, to the revised data for $\Xi^-$ and $\pi^-$. The RMS values of second differences for each sample are shown in Fig. 4. It seems that the two particles are well separated, where the numbers of entries are the same as 20 000 for each $\Xi^-$ and $\pi^-$.  

3.3. Number of $\Xi^-$ hyperon-captured events with the topology of $\sigma$-stop

As mentioned above, since there is a huge amount of background for $\rho$-stop events, we applied the CS method to $\Xi^-$ hyperon candidate tracks of $\sigma$-stop events. To get the real number of $\Xi^-$ stopping, we tentatively set $\sigma_0$ and $K_s$ to 1.0 $\mu$m and 20.75, respectively, which are the best combinational values. We performed a template fitting by minimizing $\chi^2/\text{ndf}$ for the revised Geant4 data for $\Xi^-$ and $\pi^-$ to our experimental data for 695 tracks by changing the ratio of $\Xi^-$ to $\pi^-$, where ndf is the number of degrees of freedom. However, the revised Geant4 data suggest that these low-energy scattering data should be reproduced by the only optical potential to provide a minimal difference from that used for pionic atom analysis [9]. Our experimental data have a bigger tail than that of the revised Geant4 data in the large RMS of second differences. To be free from such differences, we change the cut region for the large RMS of second differences and also change the mixing rate of $\Xi^-$ and $\pi^-$ from the Geant4 data. Figure 5 expresses $\chi^2/\text{ndf}$ versus the ratio of $\Xi^-$ and $\pi^-$ for various cut regions. For the case of a cut region set to be over 5 $\mu$m, we found the minimum $\chi^2/\text{ndf}$ to be 0.633, where $\chi^2 = 22.1$ and ndf = 35, when we took $\Xi^-$ and $\pi^-$ at 0.62 and 0.38, respectively.

A histogram of the RMS values of second differences for $\Xi^-$ and $\pi^-$ is presented for our experimental data and the revised Geant4 data in Fig. 6. In this figure, the straight line shows the RMS of second difference distribution of the E373 data and the dashed line shows the Geant4 sample data revised by measurement errors. By applying the ratio of the number of $\Xi^-$ hyperons from 0.615 to 0.625 in steps of 0.001, the number of $\Xi^-$ hyperons under the ratio ($\Xi : \pi = 0.622 : 0.378$) with 0.629 of the minimum $\chi^2/\text{ndf}$ was obtained to be 432.3 for $\sigma$-stop events among the 695 where the
Fig. 5. Minimum $\chi^2$/ndf for various cut regions in the large RMS of second differences. Numerical values are weights of $\Xi^-$ in the revised Geant4 data.

Fig. 6. RMS of the second difference distribution of the Geant4 simulation and data.

$\chi^2$ value was 22.0. The statistical error was estimated to be 7.6 as a fitting uncertainty, by which the $\chi^2$ varies by 1 from the minimum value.

In Fig. 7, minimum values of $\chi^2$/ndf are presented for several settings of $\sigma_0$. The case of $\sigma_0 = 1.0$ gives the minimum $\chi^2$/ndf. Therefore, we take the number of $\sigma$-stop events due to at-rest capture of real $\Xi^-$ hyperons to be 432.3 ± 7.6.

In the E176 experiment, the contamination by $\Sigma^-$ hyperons in the Stop events of $\Xi^-$ hyperons was a not-so-small amount as a systematic error. With the Geant4 simulation, we estimated the ratio of $\Sigma^-$ stop to $\Xi^-$ stopping events to be 8.0 ± 0.3%. The ratio of the E176 result was 15.7 ± 4.8%, 12.2 $\Sigma^-$ stops among 77.6 $\Xi^-$ stopping events, which is 2.0 times larger than the result of the simulation. If we assume that the difference could be caused by insufficient knowledge of the cascade reaction producing $\Sigma^-$ and $\Xi^-$ hyperons inside emulsion nuclei, this difference can be reproduced in the E373. The Geant4 simulation for the E373 setup gave the ratio as 1.6 ± 0.2%; thus $\Xi^-$ stopping events will be contaminated by $\Sigma^-$ stops at 3.2%, which corresponds to 14.0 events among 432.3 $\sigma$-stop events. Since the momenta of the $\Sigma^-$ hyperons produced via the reaction $K^- + ^{12}C \rightarrow K^+ + \Sigma^- + X$ are smaller than those of the $\Xi^-$ hyperons, the $\Sigma^-$ hyperons produced stop in the diamond target more easily than the $\Xi^-$ hyperons.
Fig. 7. Minimum values of $\chi^2$/ndf for several $\sigma_0$. The weight of $\Xi^-$ in the revised Geant4 data was 0.622 for each $\sigma_0$. The cut regions of the large RMS of second differences are presented in the figure.

<table>
<thead>
<tr>
<th>Short track</th>
<th>Auger</th>
<th>E373 (%)</th>
<th>E176 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>25</td>
<td>(6.3 ± 1.2)</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>155</td>
<td>(36.9 ± 3.4)</td>
</tr>
<tr>
<td>No</td>
<td>Yes</td>
<td>103</td>
<td>(24.6 ± 2.7)</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>134</td>
<td>(32.3 ± 3.1)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>417</td>
<td>56.8 ± 4.6%</td>
</tr>
</tbody>
</table>

By taking the ratio of 1/2 for $\rho$- to $\sigma$-stop events [1] into account, the number of $\Xi^-$ stopping events can be $\sim$650, which is nearly ten times larger than that of E176. Since the double-$\Lambda$ hypernuclear events were clearly produced by real $\Xi^-$ hyperon capture, we checked their RMS values of the second difference. The RMS values of the $\Xi^-$ for the double-$\Lambda$ hypernuclear events introduced in Ref. [10] such as Nagara, Mikage, Demachi–Yanagi, and Hida events are 1.04, 1.18, 0.97, and 1.34, respectively. Therefore, it was found that they were well located in the region of $\Xi^-$ as shown in Fig. 6. Finally, we concluded the number of $\sigma$-stop events via at-rest capture of $\Xi^-$ hyperons to be $432.3 \pm 7.6^{+0.0}_{-14.0}$.

4. **Trapping probability**

4.1. $\Xi^-$ captured in the light and heavy nuclei

The emulsion consists of nuclear species of light and heavy elements as (C, N, O) and (Ag, Br), respectively. The emitted particles have various minimum energies by repulsive Coulomb potential for light and heavy elements. According to the detailed discussion in Ref. [1], the length for at least one track emitted from the $\Xi^-$ captured in light elements will be between 3 and 31 $\mu$m for the E373 emulsion with a typical density of 3.60 g/cm$^3$. We also checked the presence of Auger electrons at the $\Xi^-$ captured points. To check the individual events, we picked up 417 events in the region less than 1.5 of RMS of second differences, where the region could be contaminated by at most 1.2 $\pi^-$ capture events.

Table 2 summarizes the presence of short tracks and Auger electrons at the $\Xi^-$ hyperon capture point. The ratio of $\Xi^-$ capture in light to heavy nuclei was precisely obtained as $0.76 \pm 0.08$, 0.76 ± 0.08,
Table 3. Number of events with $\Lambda$ trapping by light nuclei (C, N, O) compared to the data of E176. The parenthetical values are the number of $\Xi^-$ candidates with shorter track lengths than 800 $\mu$m, although these candidates were not used in the above analyses.

<table>
<thead>
<tr>
<th>Signal</th>
<th>E373</th>
<th>E176</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-$\Lambda$ hypernucleus</td>
<td>7</td>
<td>(0)</td>
</tr>
<tr>
<td>Twin single-$\Lambda$ hypernucleus</td>
<td>2</td>
<td>(0)</td>
</tr>
<tr>
<td>Single-$\Lambda$ hypernucleus</td>
<td>28</td>
<td>(4)</td>
</tr>
<tr>
<td>$\sigma$-stop events with $E_{\text{vis}} &gt; 28$ MeV</td>
<td>88</td>
<td>(15)</td>
</tr>
<tr>
<td>Total ($\Xi^-$ trapped by light nuclei)</td>
<td>180</td>
<td>(32)</td>
</tr>
</tbody>
</table>

Table 4. Number of events with $\Lambda$ trapping by heavy nuclei (Ag, Br) compared to the data of E176. The meaning of the parenthetical values is the same as in Table 3.

<table>
<thead>
<tr>
<th>Signal</th>
<th>E373</th>
<th>E176</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$-stop events with $E_{\text{vis}} &gt; 160$ MeV (Double-$\Lambda$)</td>
<td>10</td>
<td>(0)</td>
</tr>
<tr>
<td>$\sigma$-stop events with $E_{\text{vis}} &gt; 28$ MeV</td>
<td>111</td>
<td>(17)</td>
</tr>
<tr>
<td>Total ($\Xi^-$ trapped by heavy nuclei)</td>
<td>237</td>
<td>(39)</td>
</tr>
</tbody>
</table>

Table 5. Trapping probabilities of $\Lambda$ hyperons by nuclear fragments from $\sigma$-stop.

<table>
<thead>
<tr>
<th>Trapping probability</th>
<th>Light nuclei (C, N, O)</th>
<th>Heavy nuclei (Ag, Br)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \Lambda$</td>
<td>$5.0 \pm 1.7%$</td>
<td>$4.2 \pm 1.4%$</td>
</tr>
<tr>
<td>At least 1 $\Lambda$</td>
<td>$69.4 \pm 8.1%$</td>
<td>$51.1 \pm 5.7%$</td>
</tr>
</tbody>
</table>

which is in good agreement with $0.73 \pm 0.21$ of the E176 result and the estimated value of $(2/3)$ by Hill [11]. The emission rate of the Auger electrons associated with $\sigma$-stop events was obtained to be $30.7 \pm 3.1\%$, which is consistent with the result of E176 ($40 \pm 7\%$) within one standard deviation error. However, our result seems slightly different from the rate ($37.8 \pm 2.1\%$) of $\Sigma^-$ capture reported in Ref. [12].

4.2. Trapping probabilities of $\Lambda$ in charged nuclear fragments via $\Xi^-$ hyperon capture

In order to find the trapping probability of at least one $\Lambda$ hyperon in $\sigma$-stop events, we count the events with visible energy release, $E_{\text{vis}}$, which is the $Q$-value for the reaction of $\Xi^- + p \rightarrow \Lambda + \Lambda$ for tracks emitted from a capture point greater than 28 MeV. We assumed the emitted tracks to be protons. For the light nuclei with short track(s), we will also count events with a fragment of a single-$\Lambda$ hypernucleus as trapping of at least one $\Lambda$ hyperon. In the case of the trapping of two $\Lambda$ hyperons, the signal is double-$\Lambda$ and twin single-$\Lambda$ hypernuclei for the $\Xi^-$ capture in light nuclei. In the capture by heavy nuclei, the events with $E_{\text{vis}} > 160$ MeV can be assigned to trap two $\Lambda$ hyperons, where this is the same condition for non-mesonic decay of the crypto-fragment of a heavy double-$\Lambda$ hypernucleus as in E176 [1]. The counted results are summarized in Tables 3 and 4. Since several single-$\Lambda$ hypernuclei were detected in the events caused by the $\Xi^-$ candidates with shorter lengths than 800 $\mu$m, some amount of events by $\Xi^-$ stopping at rest can be included in 71 ($= 766 - 695$) events.

The trapping probabilities of two $\Lambda$ hyperons for light and heavy nuclei were obtained as $5.0 \pm 1.7\%$ and $4.2 \pm 1.4\%$, respectively. Regarding the trapping of at least one $\Lambda$ hyperon, its probabilities were
69.4 ± 8.1% and 51.1 ± 5.7% for light and heavy nuclei, respectively. They are summarized in Table 5.

5. Conclusion In the emulsion of the E373 experiment at KEK, we have detected 766 σ-stop events caused by at-rest stopping of negatively charged particles, which were the candidates of Ξ− hyperons tagged as quasi-free (K−, K+) events. Among these, the constant sagitta method was applied to the tracks of 695 Ξ− candidates with lengths over 800 μm. With the use of simulated data for Ξ− and π− from Geant4, we obtained suitable values of scattering constant, Ks, and constant sagitta, σ0. Comparing multiple Coulomb scattering of the candidate tracks with simulated data for Ξ− and π−, we obtained the number of 432.3 ± 7.6 tracks for real Ξ− hyperons with a systematic error of Ξ− hyperons of 3.2%. The total number of the real Ξ− stopping events could be at least ~650, which is nearly ten times higher than that of E176.

Finally, with high statistics of σ-stop events captured by Ξ− hyperons, we obtained the trapping probabilities of two Λ hyperons captured by light (C, N, O) and heavy (Ag, Br) emulsion nuclei to be 5.0 ± 1.7% and 4.2 ± 1.4%, respectively. In the case of the probabilities for at least one Λ hyperon, these are 69.4 ± 8.1% and 51.1 ± 5.7% for light and heavy nuclei, respectively. In a new emulsion-counter hybrid experiment, J-PARC E07 [13], we expect that 100 double-Λ hypernuclear events will be detected. The constant sagitta method will be useful for the identification of not only the primary particles but also decay daughters of double hypernuclei.

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